$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$ —— (6)

Now system is in steady state.

then $\lim_{t \to \infty} P_n(t) = P_n$.

$\lim_{n \to \infty} P_n'(t) = 0$.

$\mu P_{n+1} - (\lambda+\mu)P_n + \lambda P_{n-1} = 0$ —— (7)

$-\lambda P_0 + \mu P_1 = 0$ —— (8)

eq^n (7) and (8) are called steady state eq^n of the system.

Now to solve the steady state eq^n (7) and (8).

from (8)

$$\boxed{P_1 = \frac{\lambda}{\mu} P_0} \quad \text{But } \rho = \frac{\lambda}{\mu}.$$

$$\boxed{P_1 = \rho P_0} \text{ —— (9)}$$

Put $n=1$ in eq^n (7)

$\mu P_{n+1} - (\lambda+\mu) P_n + \lambda P_{n-1} = 0$

$\mu P_{1+1} - (\lambda+\mu) P_1 + \lambda P_{1-1} = 0$

$\mu P_2 - (\lambda+\mu)P_1 + \lambda P_0 = 0$

$P_2 = (1 - \frac{\lambda}{\mu}) P_1 - \frac{\lambda}{\mu} P_0$

$P_2 = (1+\rho)\rho P_0 - \rho P_0$.

$$\boxed{P_2 = \rho^2 P_0}$$

similarly $P_3 = \rho^3 P_0$

$\vdots$

$P_n = \rho^n P_0$ —— (10)

Now to find P_0

we know $\sum_{n=0}^{\infty} P_n = 1$

$P_0 + P_1 + P_2 + P_3 + \dots = 1$.

$P_0(1 + \rho + \rho^2 + \rho^3 + \dots \infty) = 1$.

= G.P.

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\rho}$

$= \frac{P_0}{(1-\rho)} = 1$.

$= P_0 = (1-\rho) \Rightarrow P_0 = \left[\frac{\mu-\lambda}{\mu}\right]$ —— (11) This is the probability of no arrival.

from eq^n (10)

$P_n = \rho^n(1-\rho)$ —— (12)

This is the probability of 'n' arrival.

Now to find the probability (queue size $\geq N$)

$= \sum_{n=N}^{\infty} P_n(t) = \sum_{n=0}^{\infty} P_n - \sum_{n=0}^{N-1} P_n$

$= 1 - [P_0 + P_1 + P_2 + \dots + P_{n-1}]$

~~= 1 - [1 +~~

$= 1 - P_0 [1 + \rho + \rho^2 + \dots + \rho^{N-1}]$

$= G.P \quad , \quad S_n = \frac{a(1-r)^2}{1-r}$.

$= 1 - P_0 \left[\frac{1-\rho^N}{\rho} \right] = 1 - \frac{(1-\rho)(1-\rho^N)}{(1-\rho)}$.

$= 1 - (1-\rho^N) = 1 - 1 + \rho^N = \rho^N$.

This is the probability when queue size $\geq N$.

Now to find $E(L_s)$ expected line length in the system.

$$E(L_s) = \sum_{n=0}^{\infty} n P_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n$$

$$= \rho(1-\rho) \sum_{n=0}^{\infty} n \rho^{n-1}$$

$$= \rho(1-\rho) \{1 + 2\rho + 3\rho^2 + 4\rho^3 + \dots + \infty\}.$$

$\left[\because (1-x)^{-n} = 1 + nx + \frac{n(n+1)x^2}{n!} \right]$

$= \rho(1-\rho)(1-\rho)^{-2}$.

$= \rho(1-\rho)^{-1}$

$= \frac{\rho}{1-\rho}$

The variance of queue length

$$\boxed{Var(n) = \frac{\rho}{(1-\rho)^2}}$$

Chapter-19.

Network Analysis (PERT/CPM).

Difference b/w CPM and PERT

(1) CPM means Critical Path Method.
~~The min. path~~
PERT means Project Evolutaion and review technique.

(2) CPM is active ~~as~~ oriented technique.
PERT is Event ~~as~~ oriented.

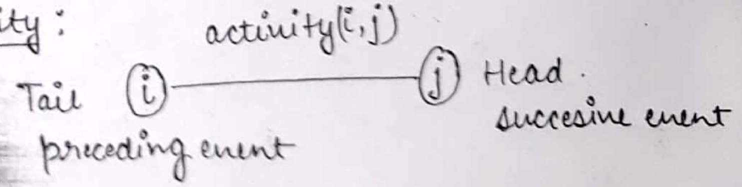
(3) CPM is deterministic model
PERT is probabilistic model.

(4) CPM is commonly used for project involving activities.
But in PERT is genrally used for one time projects involving activities and no activities is repeated.

(5) CPM model is a dual model ~~which~~ with change with time and cost but PERT model fixed for one time.

Network Components;

(i) Activity:



The activity is represented by arrow the tail of which represent the start and head is represent by finished of the activity

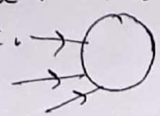
Events ; End point of activity is called event

OR

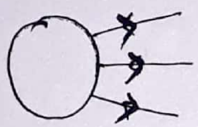
The beginning and end point of activity is called event . It is also known as 'node'

There are 3 type of events.

(a) **Merge Event** ; An event which represents the end (or head) of more than one activity is called a merge event.

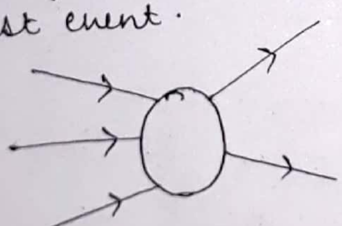


(b) **Burst Event** ; An event which represents the beginning (or tail) more than one activity is called Burst event.

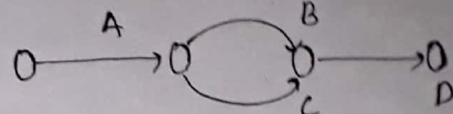


(c) **Merge and Burst Event** ;

An event which represents the beginning and the end of some activity is called merge and burst event.



Dummy Activity ;



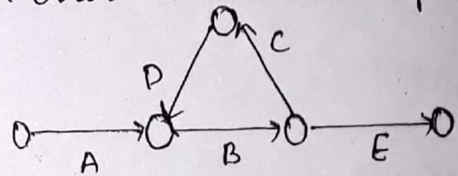
An activity which is use to represent the connection b/w events but not permanant is called dummy activity.

In fig. B, C are dummy Activity.

Errors in Drawing a Network ;

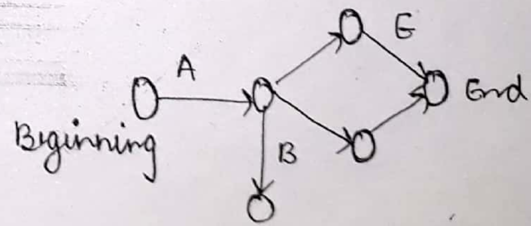
(1) **looping (or cycling) Error** ;

If some activity in the network form a loop then such error is called looping error.



BCD formed the loop. then it represent the looping error.

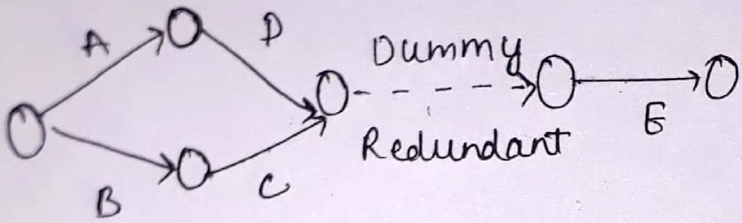
(2) **Danging Error** ; In a network some activity except final activity have not succesive activity is called danging Error.



Since Event B has not succesive event then it is danging error.

Redundancy Error :

If in network dig. the dummy activity is the only activity b/w two events is called Redundancy Error.



Ex1: Draw a network diagram for the following set of operations represented by separate letters.

Operations	Post-operations.
A	precedes B, C
B	precedes D, E
C	precedes D
D	precedes F
E	precedes G
F	precedes G

Sol^n: